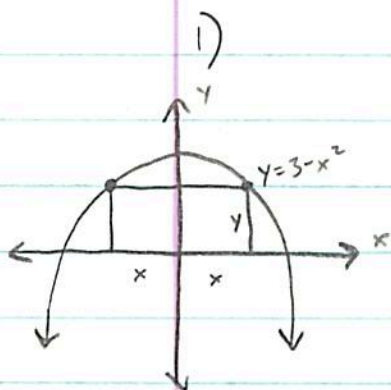


Calc Q3T1 Review Key



Maximize Area

$$A = 2xy$$

$$A = 2x(3 - x^2)$$

$$A = 6x - 2x^3$$

$$A' = 6 - 6x^2$$

A' = undefined
Never

$$A' = 0$$

$$0 = 6 - 6x^2$$

$$0 = 6(1 - x^2)$$

$$0 = 6(1+x)(1-x)$$

$$x = -1 \quad x = 1$$

reject

Restrictions

$$x > 0$$

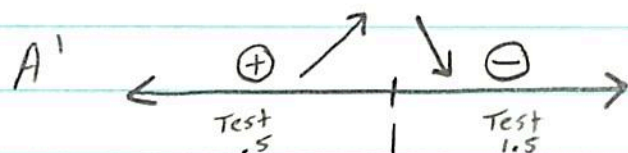
$$y > 0$$

$$3 - x^2 > 0$$

$$3 > x^2$$

$$\sqrt{3} > x$$

$$0 < x < \sqrt{3}$$



Maximum @ $x = 1$

$$A'(.5) = 6 - 6(.5)^2$$

$$= 4.5 \text{ (positive)}$$

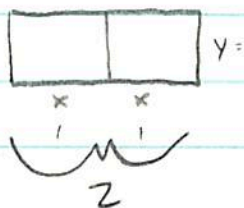
$$A'(1.5) = 6 - 6(1.5)^2$$

$$= -7.5 \text{ negative}$$

Dimensions: recall $y = 3 - x^2$

$$y = 3 - 1^2$$

$$y = 2$$



2 by 2 so Max Area = 4

2) Let $x = 1^{\text{st}} \#$

$y = 2^{\text{nd}} \#$

$$x + y = 96$$

$$y = 96 - x$$

Minimize Sum of squares

$$S = x^2 + y^2$$

$$S = x^2 + (96 - x)^2$$

$$S = x^2 + 9216 - 192x + x^2$$

$$S = 2x^2 - 192x + 9216$$

$$S' = 4x - 192$$

$S' = \text{undefined}$

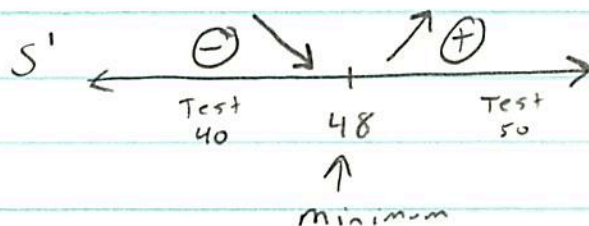
Never

$$S' = 0$$

$$0 = 4x - 192$$

$$192 = 4x$$

$$48 = x$$



$$S'(40) = 4(40) - 192$$

$$= -32 \text{ (negative)}$$

$$S'(50) = 4(50) - 192$$

$$= 8 \text{ (positive)}$$

$$x = 48$$

$$y = 96 - x = 96 - 48 = 48$$

$x = 48$	$y = 48$
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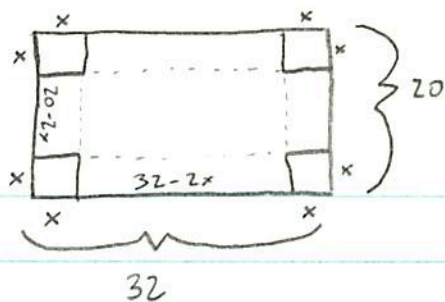
Minimum Sum

$$= 48^2 + 48^2$$

$$= 2304 + 2304$$

$$= \boxed{4608}$$

3)



Restrictions

$$x > 0$$

$$20 - 2x > 0$$

$$20 > 2x$$

$$10 > x$$

$$0 < x < 10$$

Maximize Volume

$$V = L \times W \times H$$

$$V = (32 - 2x)(20 - 2x)(x)$$

$$V = (640 - 64x - 40x + 4x^2)(x)$$

$$V = 640x - 104x^2 + 4x^3$$

$$V' = 640 - 208x + 12x^2$$

 V' = undefined

$$V' = 0$$

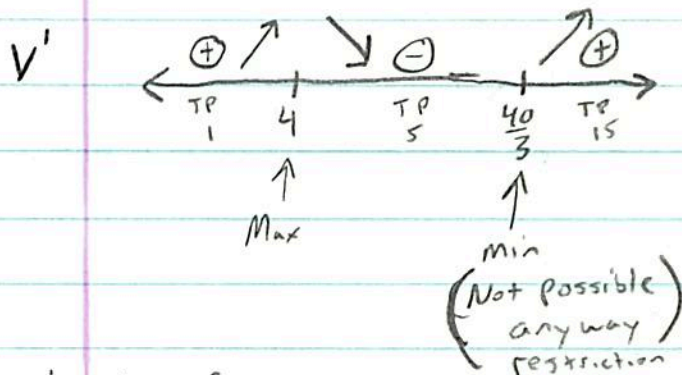
Never

$$0 = 12x^2 - 208x + 640$$

use Quad Formula

$$x = \frac{40}{3} \text{ or } x = 4$$

$$13.333$$



$$V'(1) = 640 - 208(1) + 12(1)^2$$

$$= 444 \text{ (pos)}$$

$$V'(5) = 640 - 208(5) + 12(5)^2$$

$$= -100 \text{ (neg)}$$

$$V'(15) = 640 - 208(15) + 12(15)^2$$

$$= 220 \text{ (pos)}$$

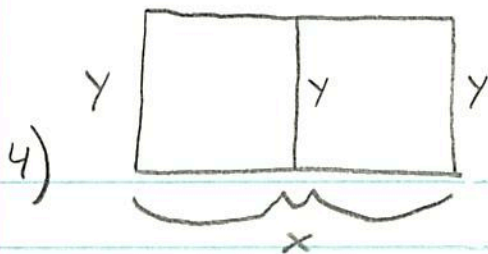
Length of squares

$$x = 4 \text{ inches}$$

Max Volume

$$V = 640(4) - 104(4)^2 + 4(4)^3$$

$$= \boxed{1152 \text{ inches}^3}$$



Minimize Perimeter

$$P = 2x + 3y$$

$$P = 2x + 3\left(\frac{6144}{x}\right)$$

$$P = 2x + \frac{18432}{x}$$

$$\text{Area} = xy$$

$$6144 = xy$$

$$\frac{6144}{x} = y$$

restrictions

$$x > 0$$

$$y > 0 \text{ so } \frac{6144}{x} > 0$$

$$P = 2x + 18432x^{-1}$$

$$P' = 2 - 18432x^{-2}$$

$$P' = 2 - \frac{18432}{x^2}$$

$$P' = \text{undefined}$$

$$x = 0$$

reject
(restriction)

$$P' = 0$$

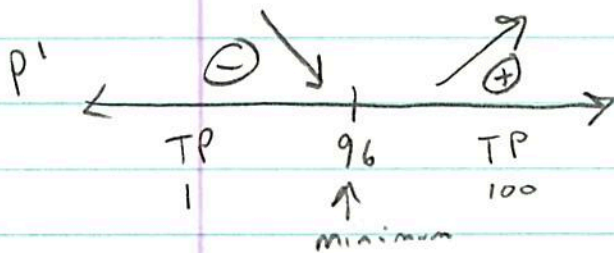
$$0 = 2 - \frac{18432}{x^2}$$

$$0 = 2x^2 - 18432$$

$$18432 = 2x^2$$

$$9216 = x^2$$

$$96 = x$$



$$P'(1) = 2 - \frac{18432}{1^2}$$

$$= 2 - 18432$$

$$= -18430 \text{ (neg)}$$

$$P'(100) = 2 - \frac{18432}{100^2}$$

$$= 2 - \frac{18432}{10000}$$

$$= 2 - 1.8432$$

$$= .1568 \text{ (pos)}$$

$$x = 96$$

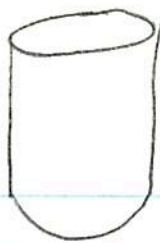
$$y = \frac{6144}{96} = 64$$

$$\text{Perimeter} = 2x + 3y$$

$$= 2(96) + 3(64)$$

$$= \boxed{384} \text{ feet of fence}$$

5)



$$V = \pi r^2 h$$

$$64\pi = \pi r^2 h$$

$$64 = r^2 h$$

$$\left(\frac{64}{r^2}\right) = h$$

Minimize Surface Area

$$SA = 2\pi r^2 + 2\pi r h$$

$$SA = 2\pi r^2 + 2\pi r \left(\frac{64}{r^2}\right)$$

$$SA = 2\pi r^2 + \frac{128\pi}{r}$$

$$SA = 2\pi r^2 + 128\pi r^{-1}$$

$$SA' = 4\pi r - 128\pi r^{-2}$$

$$SA' = 4\pi r - \frac{128\pi}{r^2}$$

SA' = undefined

 $r = 0$ reject (need a radius)

$$SA' = 0$$

$$0 = 4\pi r - \frac{128\pi}{r^2}$$

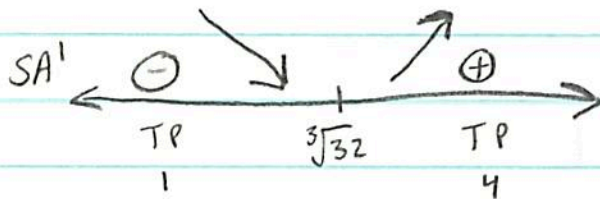
$$\frac{128\pi}{r^2} = 4\pi r$$

$$\frac{128\pi}{4\pi} = \frac{4\pi r^3}{4\pi}$$

$$32 = r^3$$

$$\sqrt[3]{32} = r$$

$$3.17 \approx r$$



$$SA'(1) = 4\pi(1) - \frac{128\pi}{1^2} = 4\pi - 128\pi = -124\pi \text{ (neg)}$$

$$SA'(4) = 4\pi(4) - \frac{128\pi}{4^2} = 16\pi - 8\pi = 8\pi \text{ (pos)}$$

Minimum SA when $r = \sqrt[3]{32} \approx 3.17$

Min SA

$$h = \frac{64}{(\sqrt[3]{32})^2} \approx 6.35$$

$$= 2\pi(\sqrt[3]{32})^2 + 2\pi(\sqrt[3]{32})\left(\frac{64}{\sqrt[3]{32}}\right)$$

$$= \boxed{189.99 \text{ in}^2}$$

$$6) C(x) = x^2 - 40x + 520$$

$$C' = 2x - 40$$

$C' = \text{undefined}$

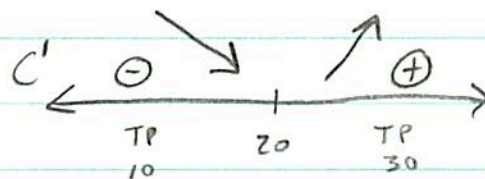
Never

$$C' = 0$$

$$0 = 2x - 40$$

$$0 = 2(x - 20)$$

$$x = 20$$



Speed = 20

$$C(20) = 20^2 - 40(20) + 520$$

Cost = \$120

$$C'(10) = 2(10) - 40 = -20$$

$$C'(30) = 2(30) - 40 = 20$$